$f : \mathbb{Z} \to \mathbb{Z}$ $\quad a, b \in \mathbb{Z}$

$$f(2a) + 2f(b) = f(f(a+b)) \quad (1)$$

nech $a+b = b+1$

$$\underbrace{f(2) + 2f(b)}_{[1;b]} = f(f(b+1)) = \underbrace{f(0) + 2f(b+1)}_{[0;b+1]}$$

$$f(2) + 2f(b) = f(0) + 2f(b+1)$$

$$\frac{f(2) - f(0)}{2} = f(b+1) - f(b)$$

$\Rightarrow$ keďže f je lineárna, $f(x) = mx + n$

dosadíme do (1)

$$m(2a) + n + 2(mb + n) = f(m(a+b) + n)$$

$$2m(a+b) + 3n = m^2(a+b) + mn + n$$

musí platit

$2m = m^2 \quad \wedge \quad 3n = mn + n$

$m(2-m) = 0$

$m = 0$ potom $3n = n$

$2n = 0$

$n = 0$

$f(x) = 0$

$m = 2$ potom $3n = 3n$

$0 = 0$

$n \in Z$

$f(x) = 2x + n$

Skúška: $f(x) = 2x + n$

$4a + n + 4b + 2n = f(2a + 2b + n)$

$4a + 4b + 3n = 4a + 4b + 2n + n$ ✓

$f(x) = 0 \qquad 0 = 0$ ✓